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INTERACTIONS OF FOUR EDGE DISLOCATIONS WITH CRACK

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ABSTRACT

The stress field is determined in an infinite nonlocal elastic solid weakened by a Griffith crack and four edge dislocations. Dislocations are located symmetrically with respect to the crack which is subject to a uniform tensile field. The problem is considered to be a plane strain problem in nonlocal elasticity.

1. INTRODUCTION

It is well-known that, classical elasticity solution of the crack and dislocation interaction possess singularities both at crack tips and at the point of application of the dislocation (cf. [1]). The non-physical nature of this solution has been debated extensively in the literature until few years ago, when the solution of the same problem, based on nonlocal elasticity, appeared (see [2]). According to the nonlocal elasticity the stress field is finite at crack tips [3,4] and at the dislocation core [5,6] but possess maxima near these points. Thus, a plastic region constructed by means of an appropriate distributions of dislocations, will not contain any stress singularity [7].

According to physics of solids, the initiation of fracture is attributed to the maximum stress field exceeding the cohesive stress that holds atomic bonds together. Consequently, nonlocal results permit the introduction of the maximum stress hypothesis as a fracture criterion [4,8], and this is shown to give excellent agreements with atomic theories and experiments [9,10].

Motivated with these results, here we pursue to discuss, a primitive problem by means of which one can construct the solution of the small scale yielding in the forms of two inclined straight lines emanating from each crack tip. This model is an accepted one, both on the basis of experiment and classical treatments [11,12]. This model can be used to construct solutions of plastic yielding based on distributions of dislocations on areas near the crack tips. Such a model is much more realistic, and it is in accordance with experimental observations.

The solution of Mode III crack, with a line distribution of screw dislocations along the crack line, was presented in a previous work [13]. The present paper represents an extension of the same problem to the Mode I crack, with inclined edge dislocations.

2. BASIC EQUATIONS

Linear theory of nonlocal elasticity, for homogeneous and isotropic solids, (vanishing body force and inertia) is governed by Cauchy's equations of motion

$$t_{kl,k} = 0 \tag{1}$$

and the stress constitutive equations, (cf. [2,9]),

$$t_{kl} = \int_V \alpha(|\underline{x}' - \underline{x}|)\, \sigma_{kl}(\underline{x}')\, dv(\underline{x}') \tag{2}$$

where σ_{kl} is given by

$$\sigma_{kl} = \lambda\, u_{r,r}\, \delta_{kl} + \mu(u_{k,l} + u_{l,k}) \tag{3}$$

Here $t_{kl}(\underline{x})$ is the real stress tensor at the reference point $\underline{x}$. It is influenced by the strains at *all points*, $\underline{x}'$, of the body, with volume V enclosed within surface ∂V. λ and μ are the usual Lamé constants and $u_k(\underline{x}')$ is the displacement vector at $\underline{x}' \epsilon V$. The influence function $\alpha(|\underline{x}' - \underline{x}|)$ depends on the distance $|\underline{x}' - \underline{x}|$ of $\underline{x}'$ from $\underline{x}$. Various forms of this function was given by Eringen [14]. Here we use

$$\alpha(|\underline{x}|) = (2\pi\varepsilon^2)^{-1} K_0(\sqrt{\underline{x} \cdot \underline{x}}\, / \,\varepsilon) \tag{4}$$

which is appropriate to the plane strain problem. Here K_0 is the modified Bessel's function of the first kind, and ε is an internal characteristic length appropriate to material; e.g., for perfect crystals it can be taken as the lattice parameter. For other kernels in 1, 2 and 3 dimensions see [14].

It is of interest to note that (4) is the Green's function of the linear operator $L = 1 - \varepsilon^2\nabla^2$, i.e.,

$$(1 - \varepsilon^2\nabla^2)\,\alpha(|\underline{x}' - \underline{x}|) = \delta(|\underline{x}' - \underline{x}|)$$

where ∇^2 is the two-dimensional Laplacian operator and $\delta(\underline{x})$ is the Dirac delta measure. This feature of the function K_0 allows us to invert the constitutive equations (2) of nonlocal elasticity:

$$(1 - \varepsilon^2\nabla^2)\, t_{kl} = \sigma_{kl} \tag{5}$$

With this apparatus at hand, Eringen [15] gave the solution of the problem of the stress field due to edge dislocations which is used in the following analysis.

3. FORMULATION OF THE PROBLEM

The main purpose of this study is to describe the stress field in an infinite elastic plane, weakened by a line crack and has four single dislocations, which are located symmetrically with respect to the crack, Fig. 1. The crack is subject to a constant tensile stress t^0 and the Burger's vectors of the four edge dislocations are:

$$-b(\cos\alpha, \sin\alpha, 0), \quad -b(-\cos\alpha, \sin\alpha, 0) \tag{6}$$

$$-b(\cos\alpha, -\sin\alpha, 0), \quad -b(-\cos\alpha, -\sin\alpha, 0)$$

The stress field is determined by the superposition of three primitive stress fields in the medium:

(i) Stress field due to the constant traction on the crack surface (t^0_{kl})

(ii) Stress field due to the dislocations (t^D_{kl})

(iii) Stress field due to the interaction of crack with dislocation (t^I_{kl})

The solution of problem (ii) was given by Eringen [15]. The stress field due to an edge dislocation with Burger's vector $b(1, 0, 0)$, and located at the origin of the coordinates, can be expressed as follows:

$$t^D_{xx}(x, y) = -\kappa\{3f_1(\rho)\sin\theta + f_3(\rho)\sin 3\theta\}, \tag{7}$$

$$t^D_{yy}(x, y) = -\kappa\{f_1(\rho)\sin\theta - f_3(\rho)\sin 3\theta\},$$

$$t^D_{xy}(x, y) = -\kappa\{f_1(\rho)\cos\theta + f_3(\rho)\cos\theta\}$$

where

$$\kappa = \frac{\mu b}{4\pi(1-\nu)\varepsilon}\,, \quad \rho = \frac{r}{\varepsilon} \tag{8}$$

$$f_1 = \frac{1}{\rho} - K_1(\rho)\,, \quad f_3(\rho) = K_3(\rho)\int_0^{\rho} I_3(\rho')\,d\rho' + I_3(\rho)\int_{\rho}^{\infty} K_3(\rho')\,d\rho'$$

Here $r = (x^2+y^2)^{\frac{1}{2}}$ and θ are polar coordinates, I_m and K_m stand for the modified Bessel's functions of first and second kinds of m^{th} order.

The problems (i) and (iii) can be combined and solved together. To this end we need to describe the load on crack surface imposed by dislocations. Considering the geometry of the problem (Fig. 1) this load can be expressed as follows:

$$\sigma^{l} = t^{*}_{xx}\sin^2\alpha + t^{*}_{yy}\cos^2\alpha + 2t^{*}_{xy}\cos\alpha\cdot\sin\alpha \tag{9}$$

where

$$t^{*}_{xx} = 2[t^{D}_{xx}(\rho_1,\theta_1) + t^{D}_{xx}(\rho_3,\theta_3)] \tag{10}$$

$$t^{*}_{yy} = 2[t^{D}_{yy}(\rho_1,\theta_1) + t^{D}_{yy}(\rho_3,\theta_3)]$$

$$t^{*}_{xy} = 2[t^{D}_{xy}(\rho_1,\theta_1) + t^{D}_{xy}(\rho_3,\theta_3)]$$

$$\begin{aligned}\rho_1 &= [(c-x)^2+a^2+2a(c-x)\cos\alpha]^{\frac{1}{2}}\,,\ \theta_1 = \pi+\psi_1-\alpha\,,\ \psi_1 = \arcsin\left[\frac{a}{\rho_1}\sin\alpha\right]\\ \rho_3 &= [(c+x)^2+a^2+2a(c+x)\cos\alpha]^{\frac{1}{2}}\,,\ \theta_3 = \pi+\psi_3-\alpha\,,\ \psi_3 = \arcsin\left[\frac{a}{\rho_3}\sin\alpha\right]\end{aligned} \tag{11}$$

To solve a crack problem which is loaded arbitrarily we follow Eringen [16]. The stress and displacement field for plane strain problems can be expressed as follows:

$$t_{xx} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{d\bar{\varphi}(\xi, y)}{dy^2} e^{-i\xi x} d\xi \tag{12}$$

$$t_{yy} = \frac{-1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \xi^2 \bar{\varphi}(\xi, y) e^{-i\xi x} d\xi$$

$$t_{xy} = \frac{i}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \xi \frac{d\bar{\varphi}(\xi, y)}{dy} e^{-i\xi x} d\xi$$

$$u(x,y) = \frac{i}{2\mu} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \xi^{-1} \left\{ \nu\xi^2\bar{\varphi}(\xi, y) + (1-\nu) \frac{d^2\bar{\varphi}(\xi, y)}{dy^2} - \varepsilon^2 \left[\nu\xi^4\bar{\varphi} + (2\nu-1) \frac{d^2\bar{\varphi}}{dy^2} + (1-\nu) \frac{d^4\bar{\varphi}}{dy^4} \right] \right\} e^{-i\xi x} d\xi$$

$$v(x,y) = \frac{1}{2\mu} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \xi^2 \left\{ (\nu-2)\xi^2 \frac{d\bar{\varphi}}{dy} + (1-\nu) \frac{d^3\bar{\varphi}}{dy^3} - \varepsilon^2 \left[(\nu-2)\xi^4 \frac{d\bar{\varphi}}{dy} + (2\nu-3)\xi^2 \frac{d^3\bar{\varphi}}{dy^3} + (1-\nu) \frac{d^5\bar{\varphi}}{dy^5} \right] \right\} e^{-i\xi x} d\xi$$

where

$$\bar{\varphi}(\xi, y) = [A(\xi) + y\, D(\xi)]\, e^{-|\xi| y} + C(\xi)\, e^{-\sqrt{\xi^2+\varepsilon^{-2}}\, y} \qquad y \geq 0 \tag{13}$$

The unknown function $A(\xi)$, $D(\xi)$ and $C(\xi)$ will be determined by the boundary conditions for specific problem under consideration.

For a crack located at $y = 0 \quad |x| < c$ and loaded symmetrically with normal tractions, i.e.

$$t_{yy} = -\sigma_0(x) = \sigma^I(x) - \sigma^0, \quad (\sigma_0(x) = \sigma_0(-x)) \tag{14}$$

we have the boundary conditions

$$t_{xy}(x, y) = 0, \quad y = 0 \tag{15}$$

and

$$v(x,y) = 0, \quad y = 0 \quad |x| > c \tag{16}$$

The boundary condition (15) yields

$$D(\xi) = |\xi| A(\xi) + \sqrt{\xi^2 + \varepsilon^{-2}}\, C(\xi) \tag{17}$$

Moreover, to ensure that t_{xx} is finite at the tip of the crack we set

$$C(\xi) = \xi^2(\xi^2 + \varepsilon^{-2})^{-\frac{1}{2}} [(\xi^2 + \varepsilon^{-2})^{\frac{1}{2}} - 2\xi]^{-1} A(\xi) \tag{18}$$

With the help of (17) and (18), the boundary conditions (14) and (16) read

$$\sqrt{\frac{2}{\pi}} \int_0^\infty \xi B(\xi)[1 + K(\varepsilon\xi)] \cos(x\xi)\, d\xi = -\sigma_0(x) \quad |x| < c \tag{19}$$

$$\int_0^\infty B(\xi) \cos(x\xi)\, d\xi = 0 \quad |x| > c \tag{20}$$

Here

$$B(\xi) = \xi \frac{\sqrt{(\varepsilon\xi)^2 + 1}\,[1 - \nu + (\varepsilon\xi)^2] + (\varepsilon\xi)[-\frac{3}{2} + \nu - (\varepsilon\xi)^2]}{\sqrt{(\varepsilon\xi)^2 + 1} - 2(\varepsilon\xi)} A(\xi) \tag{21}$$

$$K(u) = K_1(u)[1 + K_2(u)] + K_2(u) \tag{22}$$

$$K_1(u) = \frac{u^2}{\sqrt{u^2+1}\,(\sqrt{u^2+1}-2u)}\,, \quad K_2(u) = \frac{\sqrt{u^2+1}\,(\nu - u^2) + u(\frac{3}{2} - \nu + u^2) - 2u}{\sqrt{u^2+1}\,(1-\nu+u^2) + u(-\frac{3}{2}+\nu-u^2)} \tag{23}$$

After the dual integral equation, given by (19) and (20), are solved, (i.e., $B(\xi)$ is described so that the equations (19) and (20) are satisfied) the stress and the displacement fields are found from (12) taking into account the relations (17) and (18).

4. SOLUTION OF THE PROBLEM

It is convenient to introduce non-dimensional quantities

$$z = \frac{x}{c}, \quad \eta = c\xi, \quad \beta = \frac{\varepsilon}{c} \tag{24}$$

The dual integral equation (19) and (20) may now be expressed as

$$\int_0^\infty \eta\, B(\eta)\,[1 + K(\beta\eta)]\cos(z\eta)\, d\eta = -S(z), \qquad |z| < 1, \tag{25}$$

$$\int_0^\infty B(\eta)\cos(z\eta)\, d\eta = 0 \qquad |z| > 1, \tag{26}$$

where

$$S(z) = c^2\left(\frac{\pi}{2}\right)^{\frac{1}{2}} \sigma_0(cz) \tag{27}$$

The solution of this dual integral equation is not known. For an approximate solution, we recall the expression [17, 6.693.2]

$$\int_0^\infty \eta^{-1} J_{2k+1}(\eta)\cos(z\eta)\, d\eta = 0, \quad (k = 0, 1, 2, \cdots), \qquad |z| > 1 \tag{28}$$

Accordingly, if $B(\eta)$ is chosen as

$$B_N(\eta) = \sum_{k=0}^{N} b_k\, \eta^{-1} J_{2k+1}(\eta) \tag{29}$$

then the displacement boundary condition (26) will be satisfied. To determine the unknown coefficients b_k, we introduce (29) into (25)

$$S_N(z) = \sum_{k=0}^{N} b_k \int_0^\infty J_{2k+1}(\eta)\,[1 + K(\beta\eta)]\cos(z\eta)\,d\eta \tag{30}$$

and impose that

$$r(z) = [S(z) - S_N(z)]^2 \tag{31}$$

be a minimum. To provide economy in computer calculations, we take into account the limit behavior of the kernel $K(\beta\eta)$ in (25)

$$\lim_{\beta \to 0} K(\beta\eta) \to \frac{\nu}{1-\nu} \tag{32}$$

Computer experiments indicate that for $\beta \leq 10^{-2}$ the kernel $K(\beta\eta)$ behaves like (32) except near the crack tips. With this value of $K(\beta\eta)$ a great deal of simplifications is achieved, since

$$S_k(z) \equiv \int_0^\infty J_{2k+1}(\eta)\cos(z\eta)\,d\eta = \frac{\cos[(2k+1)\arcsin z]}{(1-z^2)^{\frac{1}{2}}}, \quad z < 1, \quad k = 0, 1, 2, \cdots \tag{33}$$

see [17, 6.671.2]. For integer values of k these functions can be expressed in the form of polynomials

$$S_k(z) = \sum_{j=1}^{k+1} A_j\, x^{2(j-1)}, \tag{34}$$

where

$$A_1 = 1 \,, \quad A_{j+1} = -\frac{[(2k+1)^2 - (2j-1)^2]}{(2j-1)(2j)} A_j \tag{35}$$

Since $r(z) \geq 0$ for every z, $r(z)$ is minimized in the following sense

$$\int_0^1 r(z)\, dz = \Delta z \sum_{i=1}^{M} r(z_i) \,, \quad z_i = \Delta z \,(i-1) \,, \quad \Delta z = \frac{1}{M-1} \tag{36}$$

5. RESULTS AND DISCUSSION

By means of the minimization process described in Section 4, the unknown coefficients $B(\eta)$ is determined. Afterwards, the stress field along the crack line is calculated by use of Eq. (18) and (12). The crack is loaded by a constant tensile load t^0 and four dislocations symmetrically located along two straight slip lines emanating from each crack tip. Each slip line makes an angle α with the crack line as shown in Fig. 1., and one edge dislocation is located along each line at a distance a from the crack tip. The crack length is $2c$ and the nonlocality parameter is ε.

Calculations have been performed for aluminum with properties:

$$\mu = 2.51 \times 10^{11} \text{ cgs}, \quad \nu = 0.3$$

$$b = 4.05 \times 10^{-8} \text{ cm}$$

In order to see the effect of the location of dislocation in one set of calculations (depicted by Figures 2-5) the distance a is varied while α is fixed, and in the other set (Figures 6 and 7) a is fixed and α is varied; i.e.

(i) $\alpha = 67°$; $a = 10^{-4}, \ 10^{-5}, \ 5 \times 10^{-5}, \ 5 \times 10^{-4}$ cm

(ii) $\alpha = 60°, \ 75°$; $a = 10^{-4}$ cm.

In calculating $B_N(\eta)$, given by Eq. (29), twelve terms ($N = 12$) gave satisfactory results. In fact, the maximum error was less than 0.1% between the applied surface load and the calculated ones, in all cases computed. The maximum error occurred very near the crack tips ($x = 0.99c$). Elsewhere, along the crack surface, boundary conditions were satisfied with much greater accuracy. Table 1 display the degree of accuracy by comparing the calculated surface load with the applied one in several points along the crack surface.

The error in boundary tractions near the crack tips is the result of the approximation (32) made in solving the dual integral equations (25) and (26). In previous work, Eringen and his co-workers have shown that such an approximation does not cause appreciable errors in the stress field. The stress field calculation uses the full value of the function $K(\beta\eta)$ in Eq. (25).

In all Figures we give contributions of dislocations and dislocation-crack interaction to the stress field, separately from the total stress field. The total stress fields acquire maxima near crack tips; just outside the crack tips. Contrary to the classical elasticity solution there is no singularity either at crack tips or at the points of application of dislocations. Consequently, a yield criterion may be set up by equating the maximum stress to the cohesive stress that hold bonds together. Such a yield criterion would be more meaningful for a distribution of dislocations along the slip lines. Such an investigation requires extensive analytical and computational efforts, extending our previous study [13] on Mode III crack problems.

Table 1.

Comparison of Applied and Calculated Surface Tractions.

	$\varepsilon = 10^{-5}$ cm., $c = 10^{-3}$ cm. $a = 5 \times 10^{-5}$ cm., $\alpha = 67°$ (Fig. 3)		$\varepsilon = 10^{-5}$ cm., $c = 10^{-3}$ cm. $a = 10^{-4}$ cm., $\alpha = 75°$ (Fig. 6)	
x/c	Applied	Calculated	Applied	Calculated
0.010	-1.35830	-1.35717	-1.35238	-1.35244
0.248	-1.36520	-1.36486	-1.35895	-1.35896
0.500	-1.39340	-1.39502	-1.38573	-1.38594
0.752	-1.49593	-1.49412	-1.47954	-1.47953
0.990	-3.75938	-3.76026	-1.80860	-1.80861

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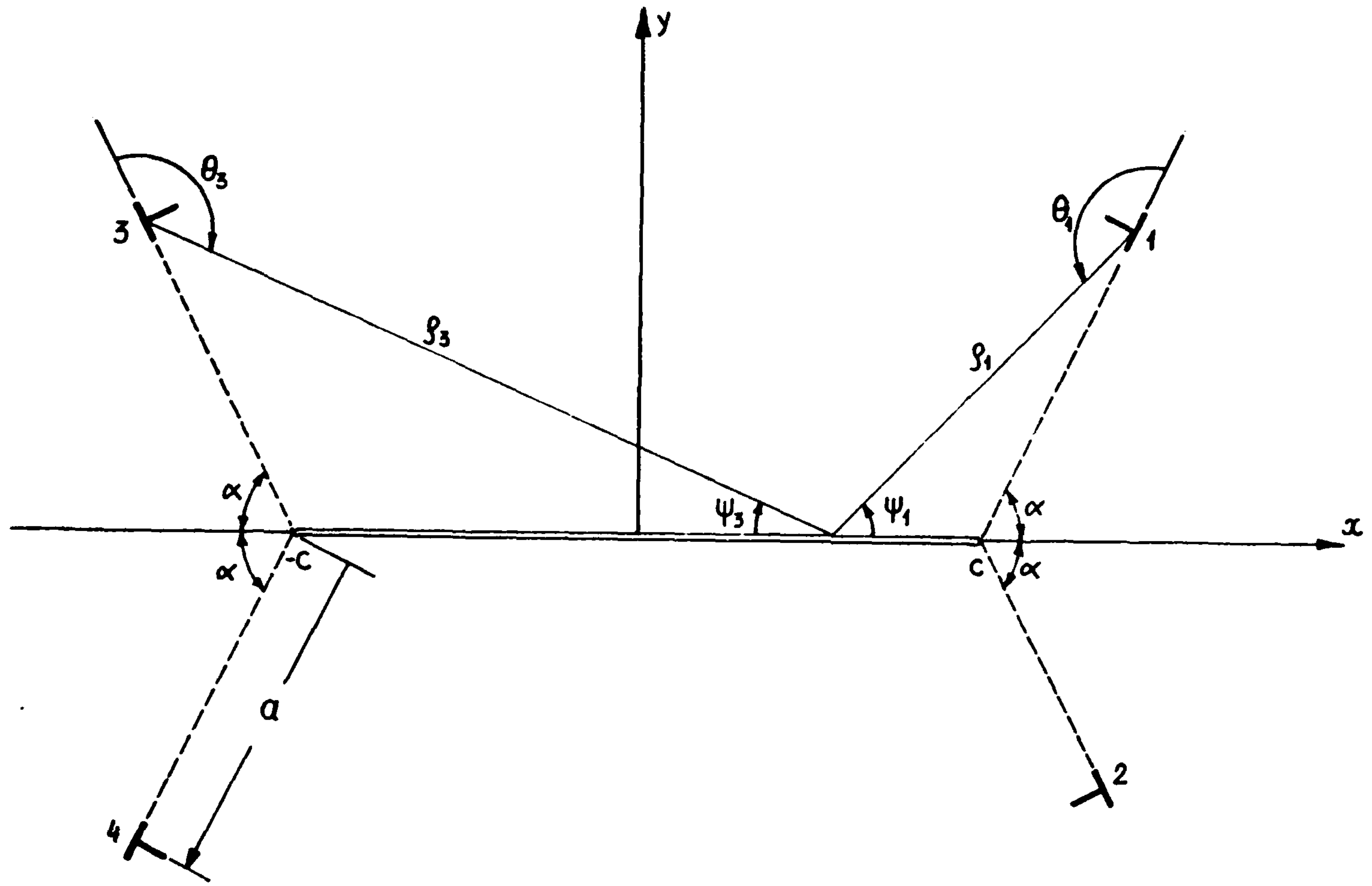

Fig. 1 Crack and four dislocations

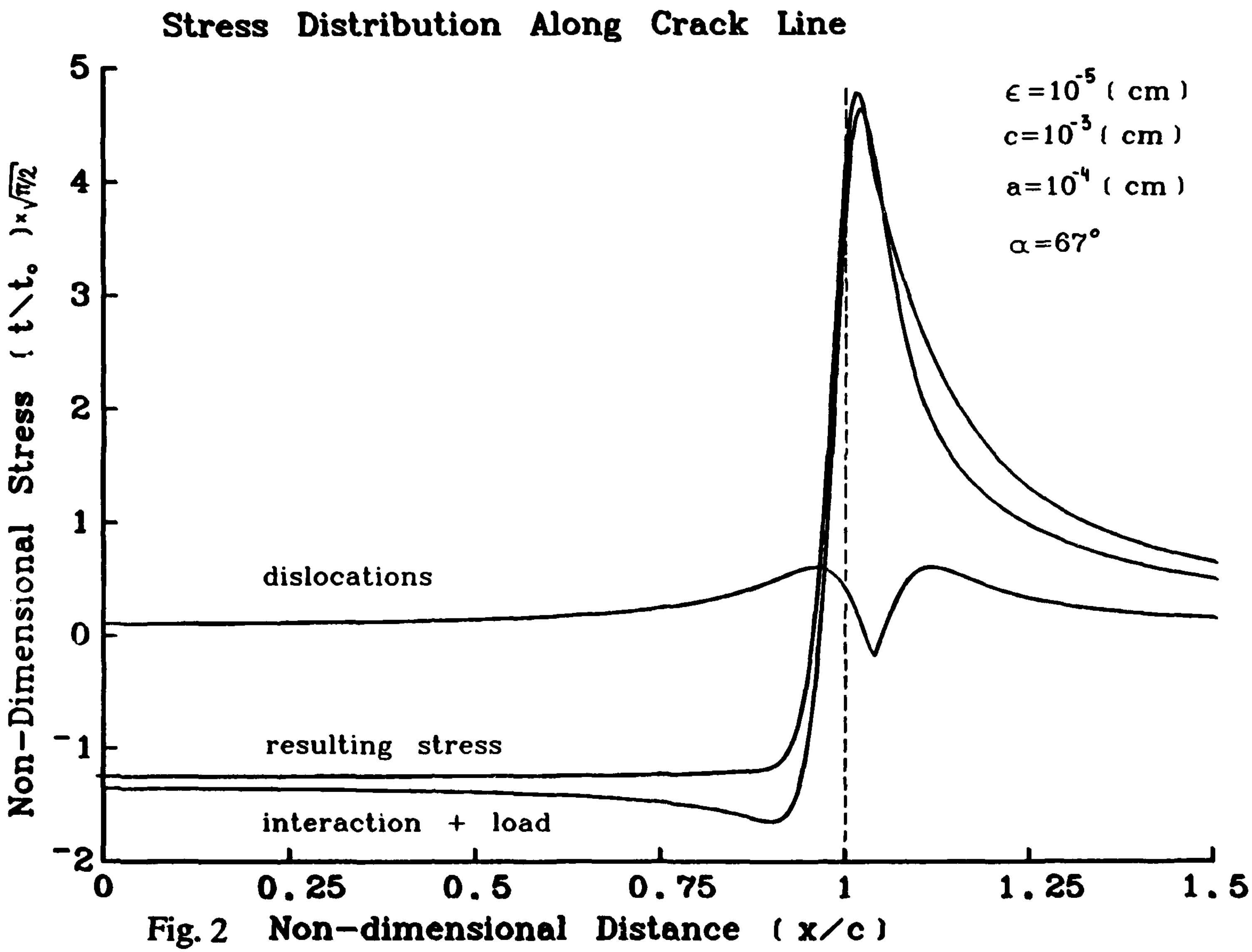
Stress Distribution Along Crack Line
$\epsilon=10^{-5}$ (cm)
$c=10^{-3}$ (cm)
$a=10^{-4}$ (cm)
$\alpha=67^{\circ}$
dislocations
resulting stress
interaction + load
Non-Dimensional Stress (t\t$_o$)$\times\sqrt{\pi/2}$
5
4
3
2
1
0
-1
-2
0
0.25
0.5
0.75
1
1.25
1.5
Non-dimensional Distance (x/c)

Fig. 2

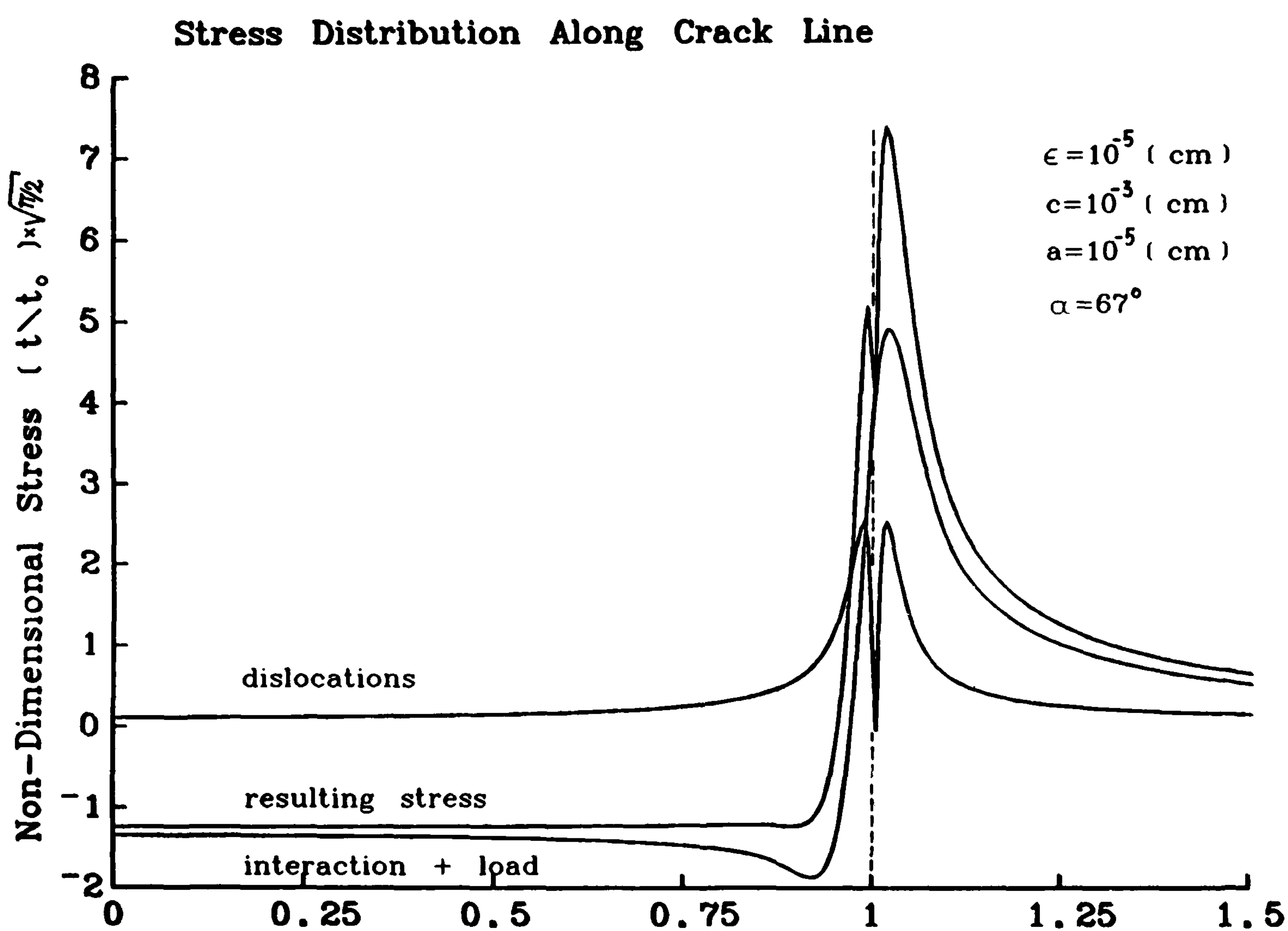

Fig. 3 Non-dimensional Distance (x/c)

Stress Distribution Along Crack Line

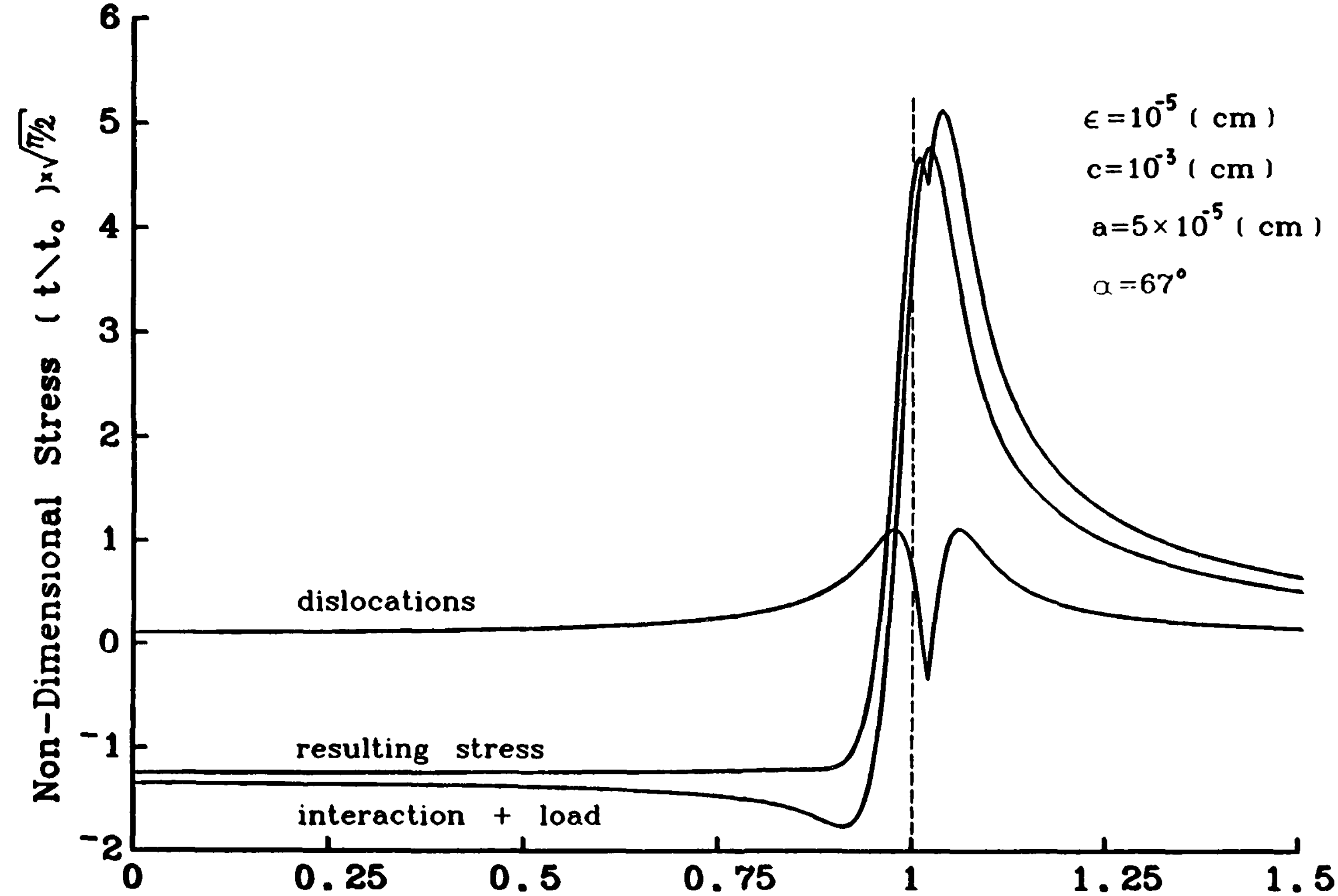

Fig. 4 Non-dimensional Distance (x/c)

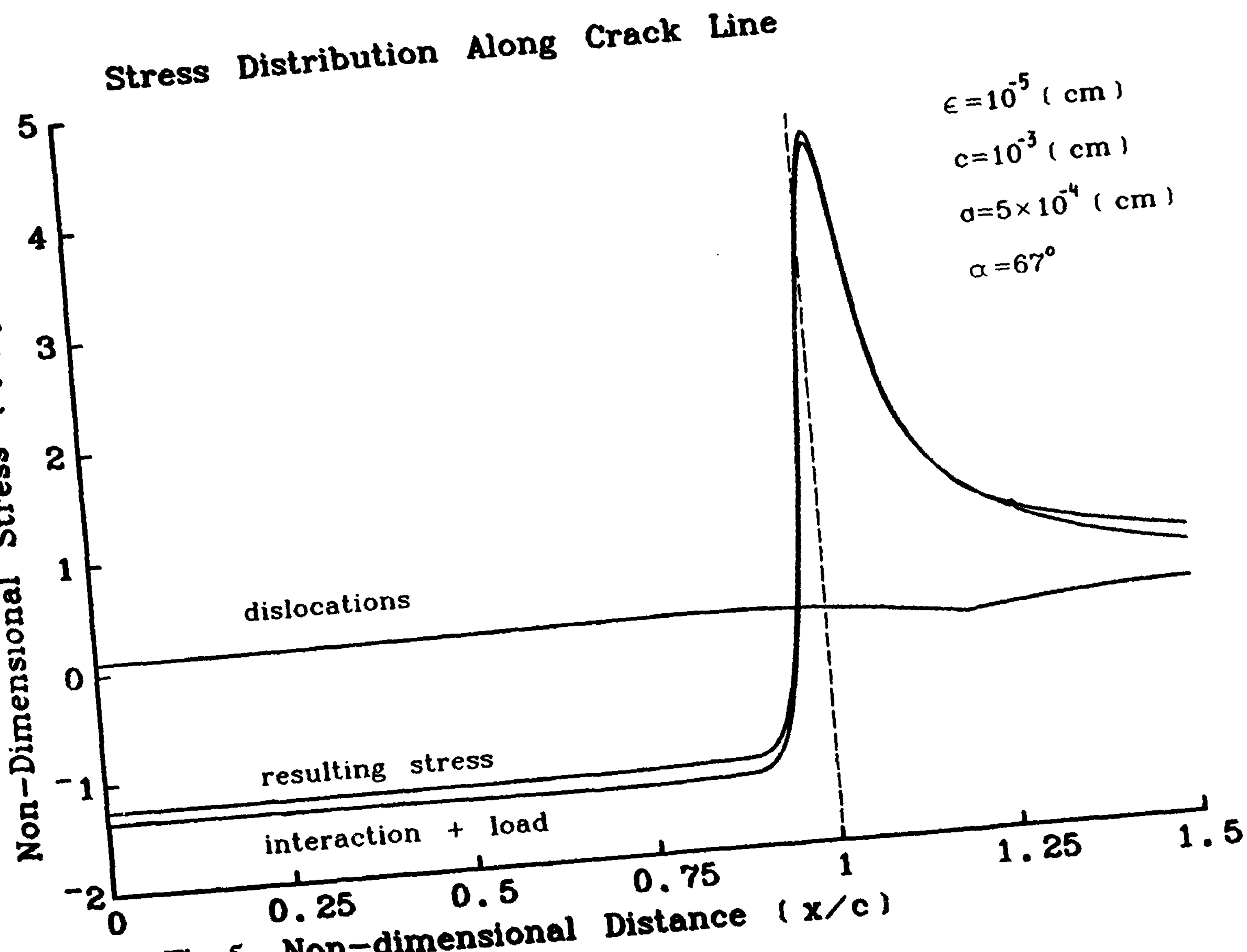
Stress Distribution Along Crack Line
$\epsilon = 10^{-5}$ (cm)
$c = 10^{-3}$ (cm)
$a = 5 \times 10^{-4}$ (cm)
$\alpha = 67^{\circ}$
dislocations
resulting stress
interaction + load
Non-Dimensional Stress (t/t_o) $\times\sqrt{\pi/2}$
5
4
3
2
1
0
-1
-2
0
0.25
0.5
0.75
1
1.25
1.5
Non-dimensional Distance (x/c)

Fig. 5

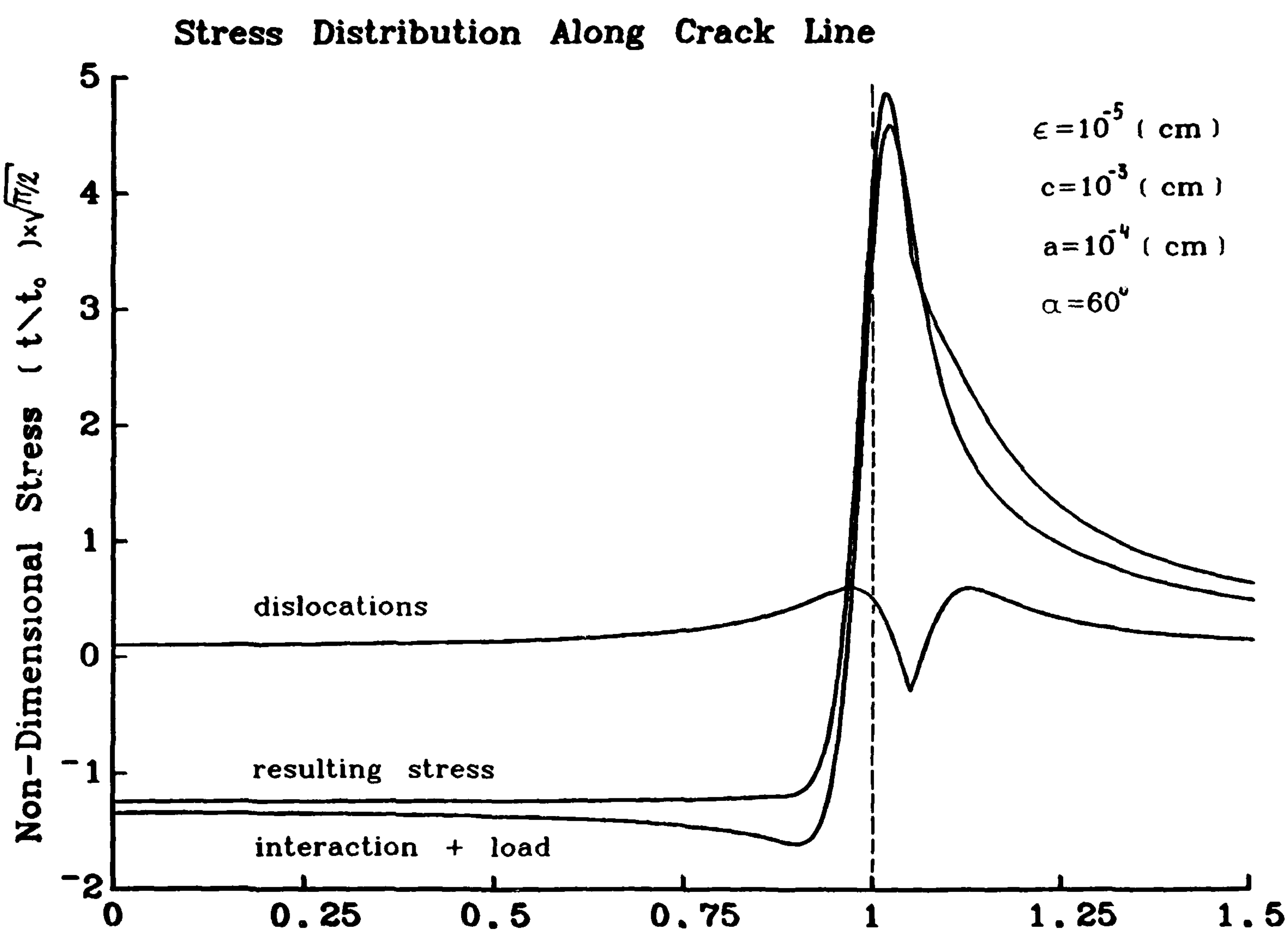

Fig. 6 Non-dimensional Distance (x/c)

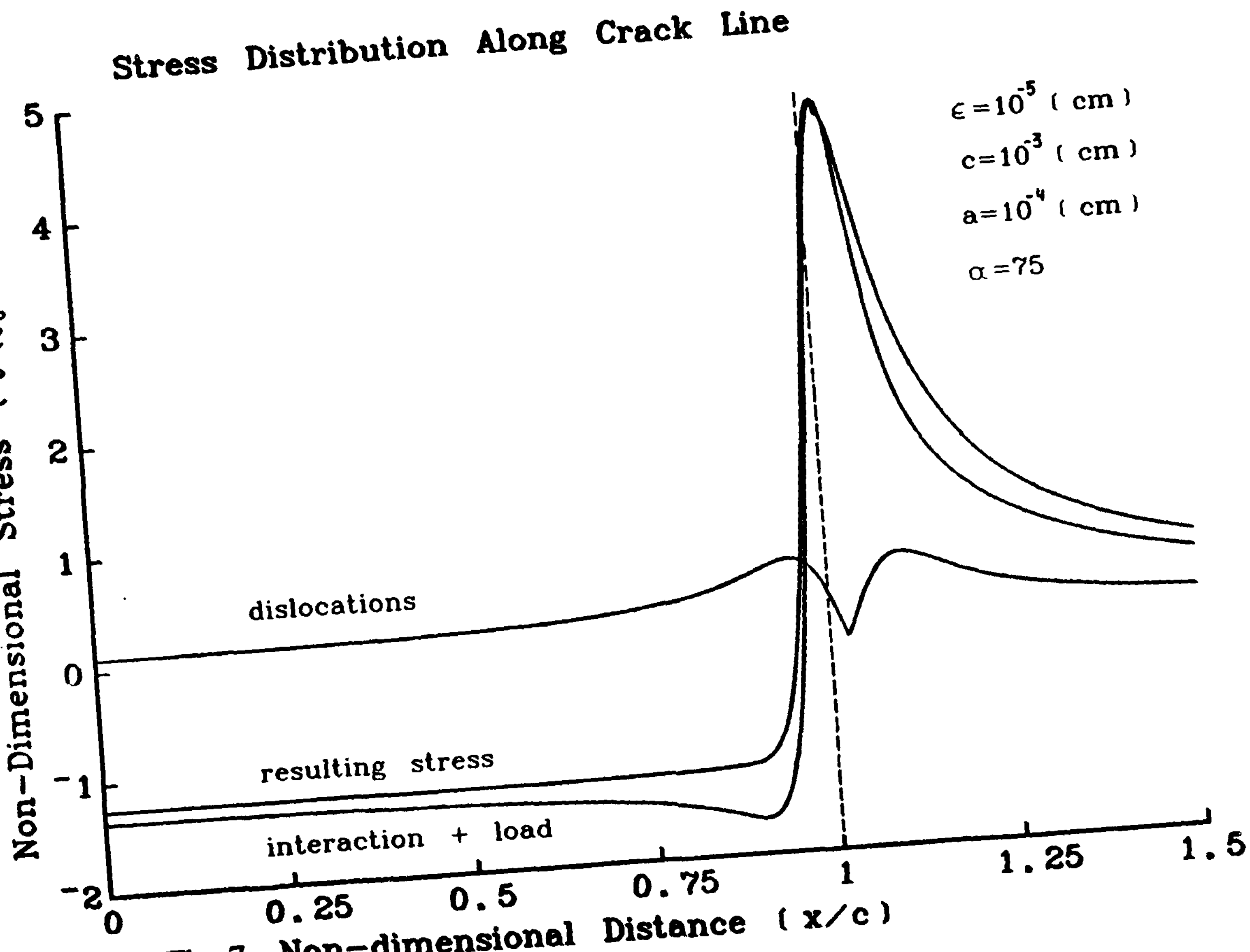
Stress Distribution Along Crack Line
ε=10^-5 (cm)
c=10^-3 (cm)
a=10^-4 (cm)
α=75
dislocations
resulting stress
interaction + load
Non-Dimensional Stress (t/t₀)×√π/2
5
4
3
2
1
0
-1
-2
0
0.25
0.5
0.75
1
1.25
1.5
Non-dimensional Distance (x/c)

Fig. 7